Social Decision-Making under Uncertainty

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Abstract

Social interactions can be deeply rewarding, yet they are marked by social uncertainty. The uncertainty arises because we can never truly know another's mind. As a species, humans find uncertainty aversive and excessive. Intolerance of uncertainty is a risk factor for emotional disorders. One way to resolve uncertainty in novel situations is to rely on memories of past similar experiences, yet this process is largely unexplored in social contexts. A novel social decision-making task showed that individuals recruit both *memories* and incremental *experiences* of social rewards to guide decisions under social uncertainty. Further, individuals with elevated depressive symptoms showed better integration of positive social memories if they also had high levels of autobiographical memory specificity. These findings suggest that successfully recalling past positive social experiences improves navigation of social uncertainty, and may confer preventative advantages for those with high levels of affective disorder symptoms.

Introduction

Uncertainty, especially in social environments, permeates daily life. Imagine you walk into a party, where you don't know anyone. You might try to decipher facial expressions and gage whether the crowd is generally welcoming or whether you might be rejected if you approach someone[1,2]. Uncertainty in social settings decreases with familiarity, but can never be fully resolved, as we can never truly know what another person thinks and feels[2]. Social uncertainty is particularly difficult to tolerate for individuals with or at-risk for depression[3], who are more likely to interpret social ambiguity negatively[4]. This intolerance of social uncertainty limits opportunities for social connection by reducing social risk-taking[3], thereby promoting the onset and maintenance of depressive states[5]. Understanding the mechanisms that modulate risk-taking under social uncertainty can inform our understanding of the aetiology of depressive symptoms and identify potentially malleable mechanisms for intervention.

We propose that one factor that exacerbates intolerance of social uncertainty in depression is reduced positive memory specificity. Reduced positive memory specificity refers to depressed individuals' difficulties in recalling specific positive events, which is combined with a tendency to have overgeneral negative memories[6]. Returning to the party example, a depressed individual would avoid talking to anyone, because their aversion of possible social rejection is potentiated by overgeneral memories of previous parties (e.g., "whenever I go to a party, I always have awkward chats with people") and difficulties in remembering specific past instances of pleasant interactions at parties. Biased episodic memory retrieval, especially in depressed individuals, then is likely to reduce individuals' ability to integrate relevant past experiences to navigate situations high in social uncertainty.

To-date, however, decision-making under uncertainty and the cognitive processes that modulate it, have been primarily studied with lab-based economic, not social, decision-making and reinforcement learning tasks[7]. Performance on these tasks relies on effective integration of past reward information. Traditionally, this process is thought to rely on error-driven learning that in effect creates a running average of experiences without remembering each individual decision[8,9]. However, these learning-based accounts of decision-making fall short in real life uncertainty, where experiences are often sparse. Emerging evidence suggests that when faced with uncertainty, people also draw on episodic memories, memories of individual events[10], to guide current decisions[11–13]. They sample from individual decision

experiences and integrate episodic memories of the outcomes from these past decision experiences to inform current decisions[12,14–17]. Economic decisions then are guided by memories of relevant past decisions.

Preliminary evidence suggests that the retrieval of past experiences also guides social decisions[18–22]. For example, using a prisoner's dilemma task, Murty et al.[19] found that participants showed more cooperation in response to previously encountered cooperative faces than previously encountered cheater faces. This adaptive cooperative behaviour was dependent on source memory of the face-reward association. Further, episodic memory for social target's characteristics has also been shown to influence adaptive approach-avoidance decisions[21]. In animal models of foraging under uncertainty, the use of episodic-like memory is observed in highly social animals[23,24]. The ability to encode and recall social information in those species may facilitate adaptive social behaviours (e.g., alliance formation in dolphins[25]). Sampling from relevant episodic memories then likely plays a critical role in forming adaptive responses under social uncertainty in humans, yet this remains largely unexplored.

In humans, the memories that guide decisions can be biased, especially in individuals who experience high levels of depressive symptoms. Depressive states are associated with altered memory processes that can modulate the integration of episodic memory in decision-making under social uncertainty in several ways. First, compared to healthy individuals, those suffering from depression show reduced episodic memory (Hedges' $g = -0.36$)[26], arguably limiting the extent to which episodic memory can be retrieved and integrated in decisions under uncertainty. Further, depression is associated with reduced access to specific positive autobiographical memory combined with a tendency toward overgeneral memory retrieval and a negative memory bias[6,27,28]. To examine whether these memory biases modulate decision-making under social uncertainty, we developed a novel social decision-making paradigm.

Building on traditional decision-making under uncertainty tasks, the paradigm translates a two-armed bandit task to a social context. The task narrative instructs participants to get as many people as possible to attend their party (i.e., accept their invitation). To invite others, participants select one of two doors to "knock" on. Each option's (i.e., door) reward probability varied according to a diffusing Gaussian random walk, with bounds at 25% and 75% (see Supplementary Table S1 for payoff probabilities on each trial). After selecting a

door, participants received immediate social feedback, with the person behind the door accepting (happy face) or rejecting (angry face) their invitation (Figure 1A). To investigate whether individuals draw on episodic memories of past events to guide current social decisions, the task presented memory probes interspersed among decision trials to remind participants of a more distant social door-outcome pairings (Figure 1B). If the probe trial successfully refreshed participants' memory of a past decision outcome, then the probed decision outcome should be integrated in the subsequent decision.

The task allowed us to test the pre-registered hypotheses (https://osf.io/7p3qj) that: participants would integrate not only the reward information from directly experienced decision trials, but also the reward information from the probed decision trial (*hypothesis 1*). Successful integration is reflected in the adoption of a win-stay-lose-shift strategy[29], where participants keep selecting doors which resulted in social acceptance, but switch door choices when a door was associated with social rejection. The integration of remembered past decision outcomes in current choices was predicted to vary with individuals' autobiographical memory specificity and depressive symptoms, where greater specificity would be positively associated with the integration of probed rewards (*hypothesis 2*) and depressive symptoms would be negatively associated with the integration of past rewards *(hypothesis 3*). Finally, the influence of depressive symptoms on social decisions was predicted to be exacerbated by reduced memory specificity. That is, the association between depressive symptoms and social decision-making would be partially accounted for by autobiographical memory specificity (*hypothesis 4*). Exploratory analyses (not pre-registered) using linear ballistic accumulator model were performed to investigate whether and how decision processes differ between experience-guided and memory-guided decisions.

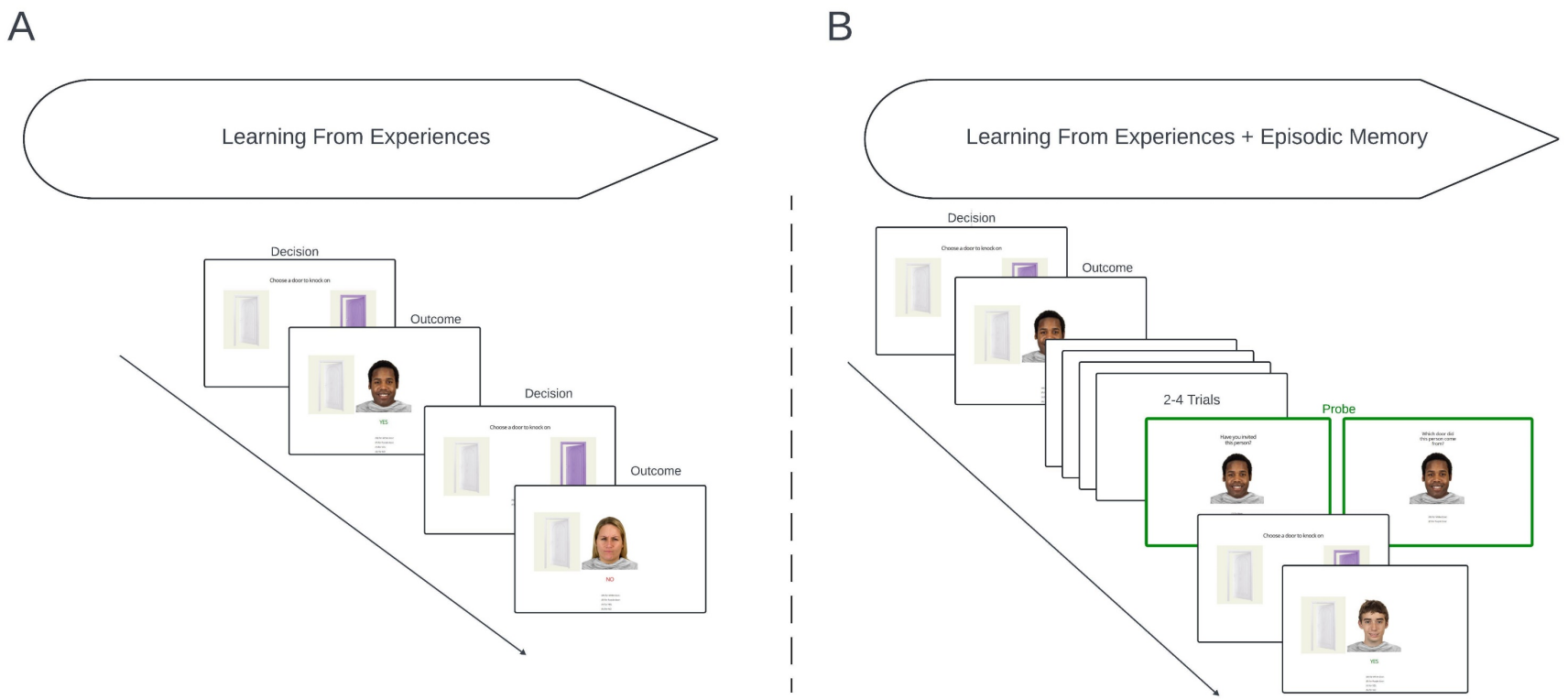

Figure 1. Social decision-making task. Figure 1A shows the decision trials with acceptance or rejection feedback. After the outcome of door choice was revealed, participants were asked to press the keyboard buttons corresponding to the selected door (W for white door, P for purple door) and result of the invitation (Y for acceptance, N for rejection) before moving on to the next trial to consolidate the memory for each decision experience. Figure 1B shows an example of a valid probe trial (highlighted in green) where participants were asked whether they have invited the person (Y for yes, N for no) and if so, which door the person came from (W for white door, P for purple door). After each probe trial, the sequence of door selections continued as before.

Results

Win-Stay-Lose-Shift in Social Decision-Making

Prior to hypothesis testing, we investigated whether participants adopted a win-stay-lose-shift strategy across all decision trials. A generalized linear mixed model showed that this was the case, with reward information from previous decision trials – up to 10 trials ago – being associated with choice on the current trial (Supplementary Table S2). For example, the odds ratio of choosing the current door was 3.48 (b = 1.25, SE = 0.04, z = 28.36, p < .001) when the party invitation was accepted by the person behind that door in the previous trial or rejected by the person behind the other door in the previous trial.

Integration of Experienced and Probed Rewards in Current Choice

In line with *hypothesis 1*, past reward information was integrated not only from directly experienced rewards, but also when the information was evoked by a memory probe (Table 1, Figure 2). Decisions immediately following memory probes were 1.49 times more likely to align with the reward information from the probed decisions (b = 0.40, SE = 0.08, z = 5.04, p < .001; Figure 2). The effect of probed reward information remained significant after controlling for individual differences in face memory ability and executive function (Supplementary Tables S13-15)

Table 1

Effects of Directly Experienced Reward and Probed Reward on Current Choice

	Lag 1					Lag 2					Lag 3				
Predictor	*Std Odds Ratio*	*b*	*SE*	*z*	*p*	*Std Odds Ratio*	*b*	*SE*	*z*	*p*	*Std Odds Ratio*	*b*	*SE*	*z*	*p*
Experienced Reward	**3.11**	**1.13**	**0.13**	**8.93**	**<.001**	**1.82**	**0.60**	**0.09**	**6.34**	**<.001**	**1.41**	**0.34**	**0.09**	**3.95**	**<.001**
Probed Reward	**1.49**	**0.40**	**0.08**	**5.04**	**<.001**	1.13	0.13	0.07	1.68	.092	1.05	0.05	0.07	0.65	.518
Probe Accuracy	0.99	-0.05	0.54	-0.10	.922	0.92	-0.55	0.44	-1.25	.213	1.05	0.28	0.47	0.60	.548
Marginal R^2 / Conditional R^2	0.079 / 0.288					0.026 / 0.159					0.008 / 0.141				

Note. Experienced reward encodes the reward outcome from 1,2, 3 decision trials ago. Probed reward encodes the reward information associated with the outcome of the probed decision trial. Probe accuracy (i.e., whether participants correctly remembered seeing the probed person and which door they came from). Experienced reward and probed reward were included in the same model at each lag.

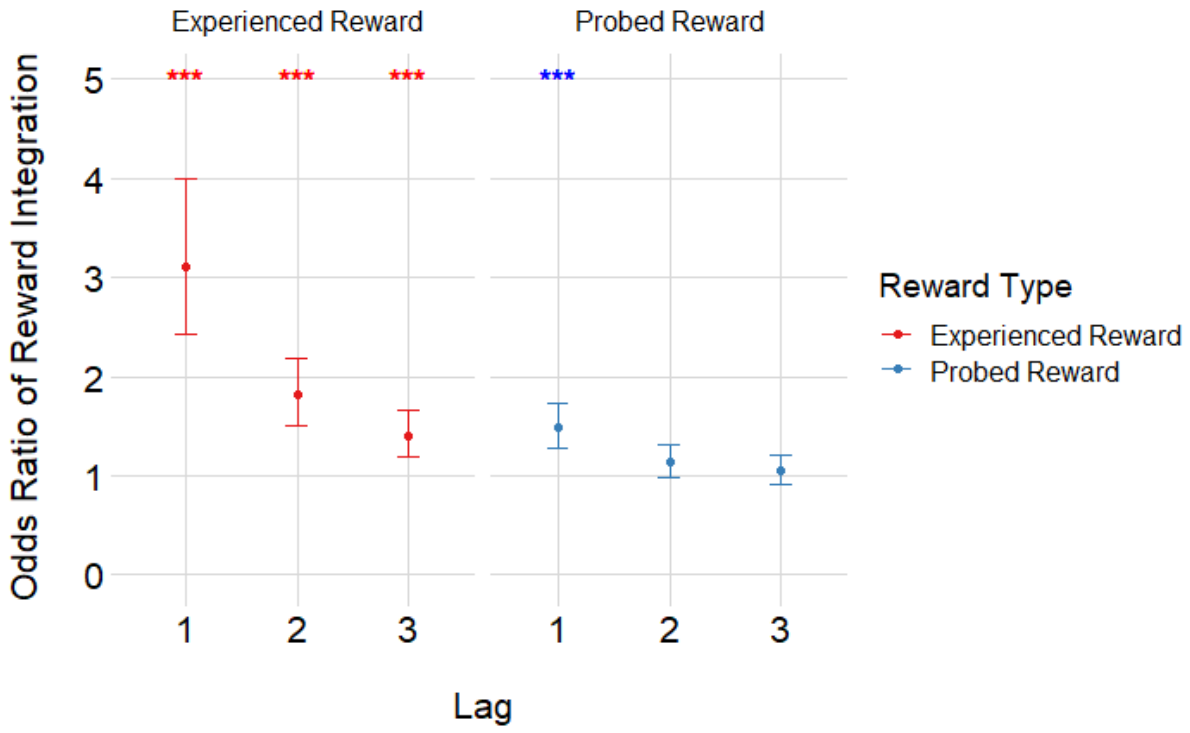

Figure 2. Main effects of past rewards. Experienced reward encodes the reward outcome. Probed reward encodes the reward information associated with the outcome of the probed decision trial. Inferences were drawn from three independent models corresponding to reward information from 1, 2, and 3 trials ago. Asterisks indicate the significance of the reward integration, at lags 1, 2 and 3: ***$p < .001$.

The pre-registered hypotheses H2 and H3 tested the main effects of autobiographical memory specificity (H2) and depressive symptoms (H3). Individually these did not significantly predict social decision-making. However, this was, in line with H4, because these effects were qualified by a higher order interaction between autobiographical memory specificity and symptoms of depression. For conciseness the main manuscript reports the interaction findings, however, the full results for H2-3 are reported in the Supplementary Information.

Integration of Acceptance versus Rejection Experiences and Memories in Decision-Making Varies with Depressive Symptoms and Memory Specificity

To examine whether the effect of depressive symptoms on social decision-making were moderated by autobiographical memory specificity, goodness of model fit was compared across Model H4a that included autobiographical memory specificity, depressive symptoms and experienced reward information, Model H4b which additionally included

probed reward information and Model H4c that additionally accounted for memory valence (i.e., acceptance/positive vs. rejection/negative). Based on AIC, Model H4c provided the best fit across all lags (Table 2) for overall memory specificity.

Table 2

Model Comparisons for H4

		AIC	BIC	χ^2	df	p
Lag 1						
	Model H4a	4254.8	4328.4	-	-	-
	Model H4b	4235.8	4434.0	26.93	4	**<.001**
	Model H4c	4191.2	4387.6	76.58	16	**<.001**
Lag 2						
	Model H4a	4521.1	4594.8	-	-	-
	Model H4b	4525.9	4624.1	3.25	4	.516
	Model H4c	4511.2	4707.6	46.69	16	**<.001**
Lag 3						
	Model H4a	4404.5	4477.7	-	-	-
	Model H4b	4410.2	4507.8	2.31	4	.679
	Model H4c	4397.2	4592.4	44.92	16	**.000**

Note. Model H4a included autobiographical memory specificity, depressive symptoms and experienced reward information as fixed effects. Model H4b additionally included probed reward information as a predictor. Model H4c additionally included memory valence as a covariate. Probe memory accuracy (i.e., whether participants correctly remembered seeing the probed person and which door they came from) was included in all models. The models were run independently for reward information from 1 trial, 2 trials, and 3 trials ago (i.e., lag 1, 2, 3).

The integration of probed reward (b = 0.21, SE = 0.10, z = 2.02, p = .043), but not rewards that had just been experienced (lag 1: b = -0.09, SE = 0.12, z = -0.77, p = .441), varied as a function of depressive symptoms and overall memory specificity (Figures 3A-B). Importantly however, as experienced rewards became more distant (i.e., lags 2 and 3) and their integration therefore relied more on memory, memory specificity also significantly modulated the integration of experienced rewards across levels of depression (lag 2: b = 0.39, SE = 0.12, z = 3.11, p = .002, Figure 3C; lag 3: b = 0.26, SE = 0.13, z = 1.96, p = .050, Figure 3E; Supplementary Table S7).

Breaking down the interaction showed that individuals did not differ in the integration of experienced social rejection information. Individuals with high (F (1, Inf) = 9.90, p = .002) and average (F (1, Inf) = 6.63, p = .010) levels of depression, however, showed better odds of integrating positive social experiences (i.e., acceptance) from 2 trials prior with increased memory specificity (Table S3, Figure 3C). There was no association between memory

specificity and integration of positive social information in individuals with low levels of depression.

Similarly, integration of probed positive, but not negative, social rewards in decisions immediately following the probe (lag 1) varied as a function of depressive symptoms and overall memory specificity (Table S3; Figure 3B). Specifically, in individuals reporting high levels of depression the integration of positive social reward information increased with overall memory specificity (F (1, Inf) = 5.02, p = .025). There was no significant effect of overall memory specificity on the integration of positive social information in individuals with low (F (1, Inf) = 0.03, p = .859) or average (F (1, Inf) = 3.81, p = .051) levels of depressive symptoms. Finally, probing memories of prior social decisions only influenced the choice immediately following the probe, but not for lags 2-3 (Figures 3D & 3F).

All associations reported here for overall memory specificity were also investigated using positive memory specificity and negative memory specificity separately. These analyses showed that the modulating effect of overall memory specificity on the associations between depressive symptoms and social memories may be driven by positive memory specificity, as the memory specificity modulation was only significant for positive memory specificity, not negative memory specificity. Full results are reported in Supplementary Information.

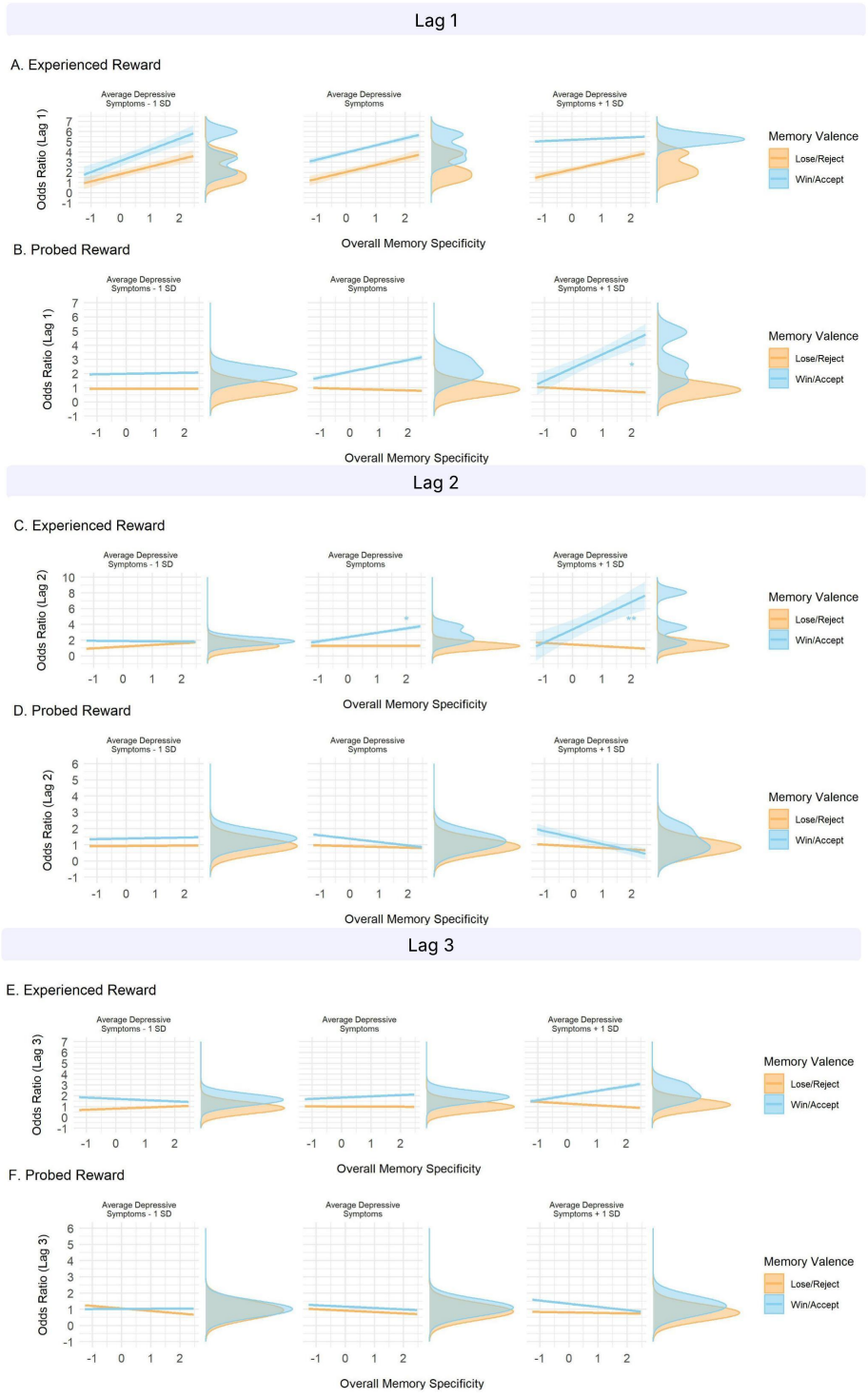

Figure 3. Integration of past rewards across levels of depressive symptoms and memory specificity. Figures 3A, 3C and 3E indicate the integration of experienced reward from 1, 2, 3 trials ago, and Figures 3B, 3D, and 3F indicate the integration of probed reward from 1, 2, 3 trials ago. Win/accept memory valence suggests that the experienced reward or probed reward information was from a win trial. Lose/reject memory valence indicates that the experienced or probed reward information was from a loss trial. Asterisks in the graph indicate the significance of the two-way interaction between reward information and positive memory specificity, at varying levels of depressive symptoms with win or loss memory valence.

Exploratory analyses: Prolonged Decision Process in Decisions after Memory Probes

Integrating reward information from a probe trial may be more cognitively demanding than integrating reward information from a directly experienced trial, as it relies on accurate mental representation of the probed reward. Prolonged reaction time was observed on decisions following probed rewards trials compared to experienced rewards trials (b = 0.40, SE = 0.01, t = 30.00, p < .001). Yet the cognitive processes underlying the prolonged decision time were unclear. We proposed that this could be due to an inflated response threshold, driving greater caution in action selection during decisions after a probe compared to those after an experience-based decision[30]. To test this account we applied a linear ballistic accumulator model[31,32], which dissociates the response time distribution (Supplementary Figure S3) and choice probability (Supplementary Figure S4) into decision-making substrates (i.e., caution (B), accumulation rate (v), starting point (A) and non-decision time ($T0$). The model that allowed the response threshold to freely vary with decision type (i.e., decisions following probed rewards vs. decisions following experienced rewards), and with accumulation rate freely varying with choice (white door vs. purple door) provided the best fit (Supplementary Table S10). Linear mixed models demonstrated greater caution in decisions requiring the integration of probed compared to experienced reward information (b = 0.68, SE = 0.04, t = 18.20, p < .001, Figure 4A). Probe accuracy modulated caution by decision type (b = 0.46, SE = 0.23, t = 1.99, p = .048), with greater probe accuracy associated with greater increases in caution on decisions after a probe (Figure 4B).

Increased memory specificity was proposed to facilitate the mental representation of the probed reward, thus reducing cognitive demands of probed reward integration; however, this was not statistically significant (b = -0.03, SE = 0.02, t = -1.34, p = .183, Supplementary Figure S5). To test whether the caution parameter simply captured the costs of task switching[33], working memory capacity was included in the model. The results showed that working memory did not modulate response caution between decision types (b = -0.00, SE = 0.01, t = -0.27, p = .789, Supplementary Figure S6), suggesting a potential mechanistic difference in decision processes guided by experience vs. memory.

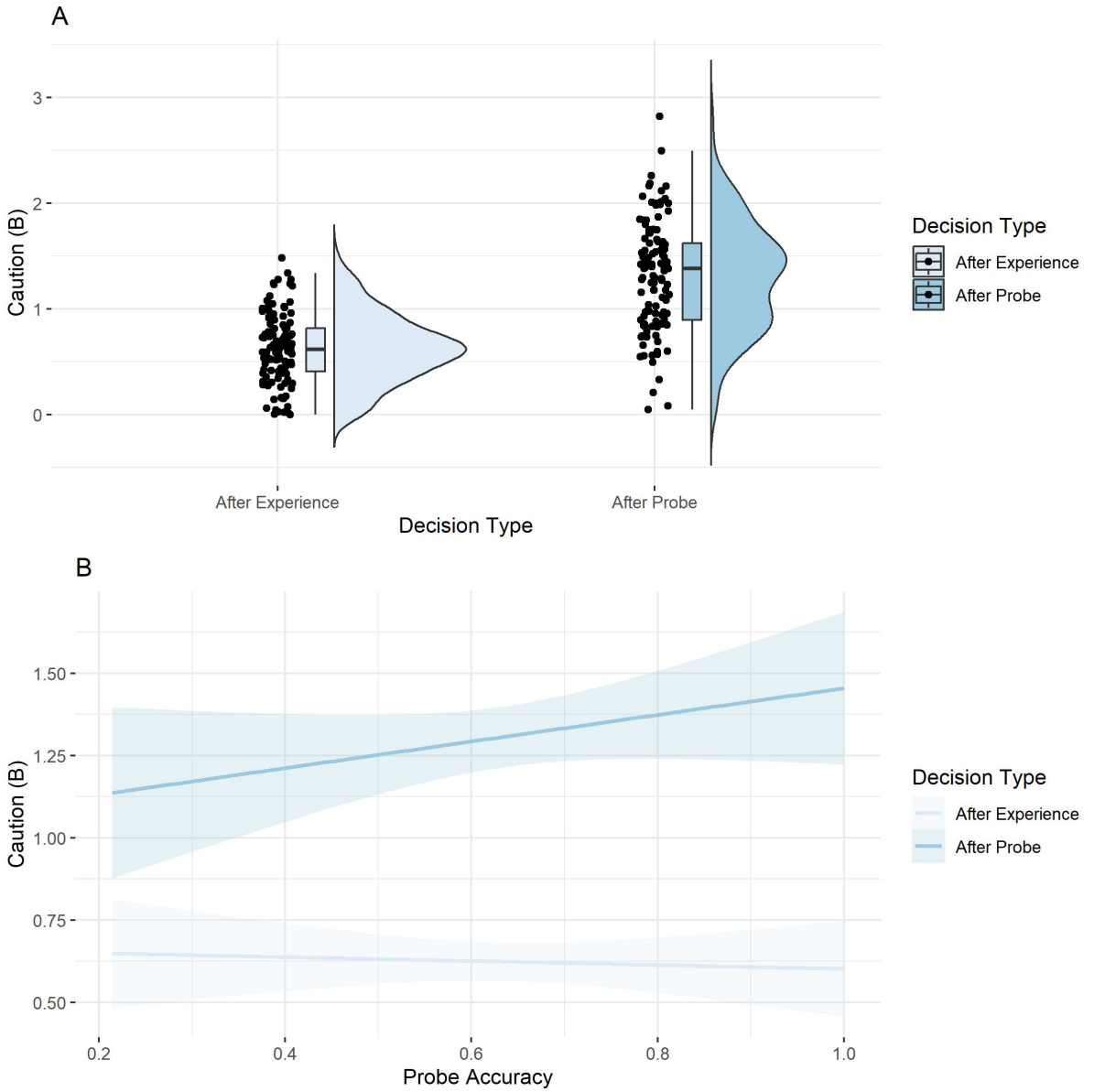

Figure 4. Response caution during social decision-making. Figure 4A shows caution by decision type. Figure 4B shows the modulating effect of probe accuracy on caution for decisions after probe trials. Caution is operationalized by the *B* parameter in the linear ballistic accumulation model. A higher value of *B* indicates greater caution. Decision type indicates whether the current decision followed a decision experience or a memory probe. Probe accuracy indicates the proportion of correctly remembered door-person associations in the 28 valid probe trials.

Discussion

Every social interaction is characterised by social uncertainty, as a person can never truly know another's mind. As a species humans find uncertainty aversive, so the mind uses heuristics to minimize exposure to uncertainty[7]. Research on economic uncertainty shows that one way to deal with the uncertainty of novel contexts is to rely on episodic memory of similar instances[14]. However, the extent to which individuals recruit memory to resolve social uncertainty is little understood. Using a novel social decision-making paradigm, this study demonstrated that much like when making decisions about non-social uncertainty[14], people also sample from past social decisions to guide current behaviour in social decision-making. The extent to which individuals sampled from memory to inform decision-making under social uncertainty varied as a function of depressive symptoms. Here individuals with high levels of depressive symptoms showed better integration of positive social *memories* and distant (i.e., lags 2 and 3) positive social *experiences* in social decision-making, but only if they had good memory specificity. The same advantage was not observed for individuals with low levels of depressive symptoms or for the integration of negative past events. Together these findings support a role of episodic memory in informing social decision-making and provide an account for the protective effects of memory specificity in guiding social decisions in individuals with high levels of depressive symptoms, especially positive memory specificity.

The effect of remembered reward information (OR = 1.49) on social decision-making was comparable to that of an experienced reward from 3 three trials ago (OR = 1.41). This finding is in line with previous research suggesting that humans enlist both incremental learning and episodic memories in value-based decision-making[13] and that episodic memories are relied upon more heavily when the current environment is more volatile[13,34]. The episodic memory-guided decision process also appears to serve as an alternative strategy as it remains intact when incremental learning is compromised (e.g., in Parkinson disease[35]). We replicated the parallel decision processes in a social context, a volatile and complex environment, demonstrating the integration of past social rewards through both incremental experiences and episodic memories. This result compliments the theoretical account of hippocampal-dependent memory as the human capacity to create rich representations of our spatial and social environments across time in service of mentally simulating possible future events and outcomes[11,36], with memory recall constituting a fictive prediction error signal that updates a person's generative model of the world[37].

Exploratory cognitive modelling of the response time data further unpacked the differences in evidence accumulation process between rewards sampled from experience vs. memory. An increase in caution (driven by inflated response threshold) on decisions after probes relative to experiences was observed. The increased caution was not simply due to task switch costs between probe trials and decision trials[33], as evident by the lack of modulation by working memory on caution. Instead, the altered evidence accumulation process appeared to be guided by the extent to which episodic memory was employed during decision-making. This was supported by the significant modulation of probe memory accuracy on caution – higher accuracy on memory probe trials was associated with greater increase in caution on decisions after probes compared to decisions after experiences. Higher probe accuracy arguably reflected a greater propensity to resort to internal mental representations of the remembered reward values. The prolonged decision process in our study was in line with previous work showing an increased decision threshold when internal memory representations of values were required during value-based decisions[38]. The exploratory findings further support the recruitment of memory in decision-making following probed memories of previous social events.

The facilitating effect of memory specificity on probed rewards integration was potentiated in individuals with high levels of depressive symptoms. The perceived volatility of social environments arguably increases as a function of depressive symptoms[39,40]. This may be due to suboptimal social learning and misestimation of uncertainty in individuals with high levels of depressive symptoms[41], especially about negative social feedback, such as the rejection cue in the current task. Indeed, individuals who experienced a volatile environment during critical period like childhood adversity showed attenuated development-typical decrease in social risk aversion[42] and persistent uncertainty about value signals[40], thereby promoting the onset and maintenance of depression. Increased perceived volatility in individuals with more depressive symptoms may have motivated these agents to rely more on episodic memory to guide decisions in the current task[13]. Future research could test this prediction by using computational methods to investigate learning rates.

The optimised reward integration in individuals with high depression and greater memory specificity, was only prominent in the context of social acceptance (i.e., wins) not social rejection (i.e., loss context). This finding adds to the mixed evidence related to hyposensitivity to rewards in depression[43–46]. The greater odds of win-stay behaviours after acceptance feedback adds to findings suggesting greater sensitivity to social rewards in

depression[47]. However, other studies using non-social reinforcement learning task have found negative[48] or no[49] associations between win-stay behaviours in individuals with depression. This suggests that these discrepancies are due to the social nature of the rewards.

Alternatively, or additionally the discrepancy in the empirical literature may be accounted for by the fact that the aforementioned tasks differ in the extent to which they require incremental learning vs. episodic memory-guided learning. Traditional (reversal) reinforcement learning/decision-making tasks are incremental in nature (e.g., no unique feature for each decision) or low in volatility (e.g., fixed reward contingencies from trial to trial). On these tasks incremental learning is sufficient for optimal performance[50]. In contrast, on tasks like the current social decision task, each decision is unique and the reward contingencies associated with the two options can be similar in value and changeable from trial to trial, mimicking the characteristics of real-world uncertain situations (i.e., episodic, complex and changeable). A combination of incremental and episodic memory learning would be a more suitable strategy for reward learning in the current task. If individuals with high levels of depressive symptoms also perceive greater volatility[40], it could further motivate people to rely more on episodic memory for decision, thus further amplifying the role of memory specificity in positive reward integration.

A further alternative and arguably complementary explanation for the valence difference in reward integration across memory specificity and depression levels could be that lose-shift can be less prominent than win-stay in learning[51]. Investigating the effect of anxiety might be a potentially promising next step given that anxiety is characterised by avoidance of threats and higher lose-shift rate in learning[52].

Importantly, the current results suggest that the potential hyposensitivity to positive reward during incremental learning in depression can be compensated by episodic memory-guided learning. The alternative memory-guided decision process in depression is dependent on autobiographical memory specificity. These findings suggest that improved navigation of social situations high in uncertainty may be one mechanism through which autobiographical memory specificity training confers preventative benefits and promotes recovery from depression. Successfully recalling past positive social experiences may encourage individuals at-risk for or with depression to approach others even when they feel uncertain, offering a chance to update their pessimistic priors, expand their social connections and gradually improve their moods.

However, while memory specificity training showed promising short-term improvement in both memory and depressive symptoms, these changes are often short-lived[53,54]. The lack of long-term and dose-sensitive therapeutic effect might suggest that episodic memory is as much a strategy as it is a cognitive capacity. Memory specificity training may simultaneously increase the capacity to access specific memories and temporarily enhance their salience, making them more likely to be used in decision-making in daily life. In the current study, memory specificity alone did not modulate the integration of reward memory. Its facilitating effect only became apparent with greater depressive symptomatology, where memory-guided decision may be preferred. The ability to access past positive events when making decisions in social situations, as well as the preference for (or partial reliance on) episodic memory are required for memory-guided decision under uncertainty. In order to change the pessimistic priors, longer reinforcement history between memory guided-decisions and positive outcomes is needed. Through this lens, memory specificity training, as a low intensity training, may be most fruitful in the prevention of depression, when pessimistic priors are less ingrained.

These findings need to be considered within the contexts of the study's limitations. Firstly, the task did not test flexible retrieval/sampling. Probes acted as memory cues that promoted memory retrieval. In real world contexts, one needs to flexibly sample from memories and select those pertinent to the current goal. Another limitation of the task is that experienced and probed rewards are not independent, though modelled separately they are overlapping, though sensitivity analyses provided strong support for the recruitment of memory processes in decisions following probed trials. The exploratory analysis may also have been limited by the unbalanced sample sizes between memory-guided and experience-guided decisions. Future research should systematically compare the qualitative differences in information sampling processes between memory-guided decisions and experience-guided decisions.

In conclusion, decision-making under social uncertainty relied on both incremental learning from repeated past experiences and episodic memory in the current study. Individuals with high levels of depressive symptoms showed better integration of positive memories if they showed good autobiographical memory specificity. Improving positive memory sampling could be a potential target for future memory training interventions.

Method

Participants and Procedures

134 participants (18-59 years, M_{age} = 33.78, SD_{age} = 10.33) were recruited via online experiment platform Prolific (https://www.prolific.com/). The study was pre-registered (https://osf.io/7p3qj) and presented on Gorilla (https://gorilla.sc/). All participants provided consent and received £9 for participation, and had the opportunity to win up to an extra £5 depending on task performance. 11 Participants were excluded due to response biases (i.e., choices were more than 90% to either option) that indicated they did not attempt to learn the reward associated with each option in the social decision-making task. 1 participant was excluded due to experiencing a technical issue while completing the social decision-making task. 122 participants were included in the analyses (Table 3).

Table 3

Participant Characteristics and Descriptives

Participant characteristics		N (%)
Age		
	18-24 years	24 (19.67%)
	25-65 years	98 (80.33%)
Gender		
	Female	69 (56.56%)
	Male	50 (40.98%)
	Non-binary	3 (2.46%)
	Prefer not to say	0
Ethnicity		
	Asian	20 (16.39%)
	Black	9 (7.38%)
	White	75 (61.48%)
	Hispanic	5 (4.10%)
	Mixed	10 (8.20%)
	Other	3 (2.46%)
	Aboriginal or Torres Strait Islander	0
	Prefer not to say	0

Subjective SES		
	Very well off	1 (0.82%)
	Rather well off	18 (14.75%)
	Fairly well off	47 (38.52%)
	Not very well off	44 (36.07%)
	Not at all well off	12 (9.84%)
Education		
	University	78 (63.93%)
	Professional/vocational training	18 (14.75%)
	High School	25 (20.49%)
	Primary School	1 (0.82%)
Questionnaires or Tasks Descriptives		Mean (SD)
Depressive Symptoms		6.02 (5.51)
Autobiographical Memory		
Specificity	Positive	1.02 (1.91)
	Negative	0.20 (1.22)
	Overall	0.61 (1.87)
Cognitive Control		
	No. correct trials	19.12 (3.64)
	Reaction time (ms)	1099.65 (272.02)
Affective Control		
	No. correct trials (affective minus neutral condition)	-0.77 (3.13)
	Reaction time (ms)	1111.25 (237.39)
Face Memory Ability		50.46 (12.00)

Note. Depressive symptoms were measured using Patient Health Questionnaire (PHQ-8) [55]. Autobiographical memory specificity was measured by Autobiographical Memory Test (AMT) [56]. Memory specificity (positive, negative or overall) was constructed by the relative ratio of specific to overgeneral memories on AMT. Positive memory specificity refers to memories in response to positive cue words. Negative memory specificity refers to memories in response to negative cue words. Overall memory specificity refers to memories in response to both positive and negative cue words. Face memory ability was measured by Cambridge Face Memory Test (CFMT) [57]. Cognitive control was measured as reaction time for correct trials and number of correct trials in the neutral condition in a 2-back task (n-back task) [58]. Affective control was operationalized as reaction time for correct responses and number of

correct trials in the affective condition minus number of correct responses in the neutral condition in the 2-back task [58].

Measures

Symptoms of Depression

Depressive symptoms were assessed by the eight-item Patient Health Questionnaire (PHQ-8)[55]. The measure has been shown to be a reliable index of depression[59]. Participants rated each item (e.g., "Feeling down, depressed, or hopeless") on a four-point scale (0 = not at all, 3 = nearly every day). Depressive symptoms were operationalized as total score on the PHQ-8. The PHQ-8 showed good internal consistency in current study, $\omega_T = 0.91$.

Social Decision-Making

The social decision-making task is a modified version of the two-armed bandit task employed by Bornstein and colleagues[14]. In the task, participants were instructed to organize a social event and get as many people as possible to accept their invitation to the event. Participants were presented with two doors to select from ("knock on"). Following their selection, they saw either a smiling face (win), which indicated acceptance of the invitation, or an angry face (loss), which indicated rejection of the invitation (Figure 1A). The probability of each door giving an acceptance response changed independently on each trial according to a diffusing Gaussian random walk with reflecting bounds at 25% and 75%. The initial payoff probabilities were set to 60% and 40% with the superior starting door counterbalanced among participants (see Supplementary Table S1 for payoff probabilities for each trial).

Participants were instructed to remember the door-person-outcome associations. Throughout the task, participants were periodically reminded of past decisions through memory probe trials (Figure 1B). In memory probe trials, the photo of a person (with either a smiling or angry expression, depending on the outcome from the probed decision) was presented. Participants indicated whether they have invited this person and if so, which door this person came from. If they indicated not having seen the person, the follow-up question asked them to indicate whether the person in the photo was wearing a purple or white t-shirt to control for time between decisions. After responding to the memory probe participants were not provided with feedback about their memory accuracy. If the probe trial successfully refreshed participant's memory of a past decision experience, then the probed decision should be more likely to be incorporated in the ensuing decision (i.e., avoid loss doors and select win doors).

The social decision-making task was presented across 4 blocks each with 32 decision trials interspersed with 8 probe trials (including, 7 valid probes and 1 invalid probe). Each block was followed by 7 post-task memory trials, where the valid probe faces were presented as neutral faces and participants were asked to indicate which door the person came from and whether they accepted the invite. 14 of the valid probe trials were probes of acceptance trials and the other 14 valid probes were of rejection trials to model the impact of memory valence on decision-making. The probed decision was from 2-4 trials prior the current trial. Performances on both probe trials and post-task memory task were above chance level (probe memory accuracy: $M = 0.63$, $SD = 0.16$; post-task memory accuracy: $M = 0.69$, $SD = 0.12$), indicating the validity of the memory probes as reminders of past decisions.

Autobiographical Memory Specificity

Autobiographical Memory Test was used to measure autobiographical memory specificity (AMT) [56]. In AMT, participants are asked to produce a specific memory in response to each valenced cue word within one minute. The valenced cue words consist of five positive ones (i.e., happy, surprised, safe, successful and interested) and five negative ones (i.e., sad, lonely, hurt, careless and angry). Autobiographical memory specificity was operationlised by the relative ratio of specific to overgeneral memories on the AMT. Similarly, positive (negative) autobiographical memory specificity was measured by the relative ratio of specific to overgeneral positive (negative) memories on AMT. Two trained coders W.W. and Y.H. independently coded each response on AMT. The inter-rater reliability for overgeneral response was very good: inter-rater agreement was 99.63%, 99.85%, and 99.85% for overall, positive and negative memory respectively; Cohen's κ was 0.99, 0.99, and 1.00 for overall, positive and negative memory respectively. All ambiguous codings were discussed at a consensus meeting of trained researchers and a coding was agreed on.

Face Memory Ability

In order to control for individual differences in face memory ability in the social decision-making task, the Cambridge Face Memory Test was administered (CFMT) [57]. Participants were given a learning phase to memorize a face within a time limit. They were then presented with three faces and one of which was the one they memorized during the learning phase. They were asked to identify the target with forced choice. There were 72 trials in total. Face memory ability was operationalized as total score on the 72 trials of the Cambridge Face Memory Test.

Cognitive and Affective Control

A 2-back task [58] was included to control for cognitive and affective control. The 2-back task included two conditions, one including neutral words (e.g., list, stair) the other affective words (e.g., good, fraud). Each condition was made up of 22 trials, where participants indicated whether the current word was the same as the word presented 2 trials ago. Presentation order of the conditions was randomized across participants, with half the participants seeing the neutral condition first, while the other half were presented with the affective condition first. Cognitive control was measured as reaction time for correct trials and number of correct trials in the neutral condition. Affective control was operationalized as reaction time for correct responses and number of correct trials in the affective condition minus number of correct responses in the neutral condition. Cognitive and affective control were included in sensitivity analyses to explore whether any observed effects of autobiographical memory specificity could be accounted for by difference in cognitive or affective control.

Analyses

All generalized linear mixed models included ID as random effect; fixed effects included in the models are specified below.

Before hypothesis testing, we first investigated whether participants followed the win-stay-lose-shift strategy using a generalized linear mixed model. It was predicted that the current choice would be positively associated with the reward experienced in previous trials.

To test hypothesis 1 that choices following a memory probe trial will not only be predicted by the directly experienced rewards but also rewards from the probed decision trials, we used a generalized linear mixed model with both experienced and probed rewards as predictors.

The second hypothesis, that autobiographical memory specificity will be positively associated with integration of the reward-contingencies on probed trials, was tested by adding autobiographical memory specificity (operationalised as ratio of specific to overgeneral memories) as a predictor to the model specified under hypothesis 1.

To test hypothesis 3 that participants' performance on the social decision-making task will be associated with depressive symptoms, generalized linear mixed models were built including depressive symptoms and: experienced rewards (model A); experienced and probed rewards

(model B) and experienced rewards and probed rewards by memory type (win/acceptance vs loss/rejection) (model C).

To test hypothesis 4 that the association between levels of depression symptoms and social decision-making performance will be partially accounted for by autobiographical memory a moderation analysis was performed.

For all models including probed reward as predictor, sensitivity analyses were conducted to confirm the effects hold when including probe memory accuracy as a covariate. Sensitivity analyses were also conducted to confirm the effects hold when including face memory ability, cognitive control and affective control ability as covariates separately. Results of sensitivity analyses were included in Supplementary Materials.

Exploratory cognitive modelling was run to investigate whether and how decision processes differ when reward information was sampled from memories compared to experiences. We jointly modelled reaction time and choice probability using the linear ballistic accumulator (LBA) model[31,32] in the dynamic models of choice package (DMC)[60]. The LBA model estimated four parameters: non-decision time (*T0*), response threshold (*b*), accumulation rate (*v*), and starting point bias (*A*). *T0* accounts for time taken by non-decision processes, such as encoding stimulus and enacting response selection; *b* is the response threshold of evidence accumulation; *v* indicates the rate of evidence accumulation; *A* determines the interval at which each accumulator begins. Together, the threshold (*b*) and starting point (*A*) are parameterised as caution ($B = b - A$) in the DMC. The parameters related to evidence accumulation processes (*B* and *v*) were of primary interest in the current analyses. Five models were built to investigate the decision processes between the decision type (i.e., after probe vs. after experience): the first model was a null model where only *v* was free to vary with choice response (select white or purple door). An additional three models were built to compare the effect of decision type on each parameter, starting point *(A)*, accumulation rate (*v*) and caution (*B*). In the fifth model, both accumulation rate and caution were free to vary with decision type, and accumulation rate was additionally modeled on choice response. The model with caution (*B*) free to vary with decision type (after probe vs. after experience), and accumulation rate (*v*) free to vary with choice response (select white door or purple door) provided the best fit using a fully Bayesian method of model comparison (the Bayesian posterior information criterion)[61]. Model comparison results were included in Supplementary Table S10. An alternative explanation for the increased reaction time in decisions after probes may be the prolonged deliberation required

when conflicting evidence is present. That is, increases in caution may be primarily driven by cases where evidence drawn from the probe reward conflicted with evidence drawn from the recently experienced reward. A sensitivity LBA model was therefore built to further allow caution to vary with the presence of conflicting evidence covariate in the best fit model. A sensitivity test was conducted to confirm that decision type had an effect over and above the presence of conflicting evidence and there was no interaction between the two. The results of the sensitivity test were included in the Supplementary Information. Linear mixed models were used to further investigate the difference in mean response caution between decision types. The caution parameter was the outcome variable, decision type was the fixed effect, and participant ID was random effect. Probe accuracy, affective control, or memory specificity was additionally entered as covariate to the mixed model.

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Contributions

T.C., and S.S. conceptualised the study. W.W. and S.S. designed the study. W.W. programmed the tasks and collected the data. W.W., G.C., and B.J.C. analysed the data. W.W. drafted the manuscript under the supervision of S.S.. G.C. and A.M.B. provided critical revisions to the manuscript. All authors contributed to and approved the final manuscript.

Data availability

The datasets used in this paper are available online at https://osf.io/7p3qj. The stimuli used in the social decision task were sourced from the Chicago Face Database (https://www.chicagofaces.org/) and the Racially Diverse Affective Expression (RADIATE) Emotional Face Stimulus Set (https://abcdstudy.org/scientists/abcd-fmri-tasks-and-tools).

Code availability

The codes used in this paper for analyses and figures are available online at https://osf.io/7p3qj.

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