Distinguishing between descriptive and rational approaches to collapsing decision bounds: a comment on Hawkins et al. (2015)

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Sequential sampling models have emerged as a standard theory of decision making in two-alternative tasks. For decades, they have been employed to recover and predict behavior and neural activity in problem domains as diverse as memory recognition, perceptual inference, and value-guided decision making (e.g. Ratcliff, 1978; Britten et al., 1992; Krajbich et al., 2010). In these models, some decision variable or variables accumulate potentially noisy evidence over time until a response threshold is reached. Supporting extensive empirical observations, analytical work has shown that one of the most successful such models, the diffusion decision model (DDM, Ratcliff 1978), is analytically equivalent to Wald's sequential probability ratio test (SPRT), the optimal sequential test between two simple hypotheses (Wald and Wolfowitz 1948; analysis by Bogacz et al. 2006). This situates the DDM not only as a descriptive model, but also as a *rational* model of decision making and as such as a candidate unifying neural computation.

The struggle to maintain this dual status creates a tension, for example in the fact that the proven-optimal "pure" DDM (pDDM) fails to recover some features of the data, such as long-tailed and unequal response- and accuracy-conditioned RT distributions. Because of this, many descriptive successes attributed to the DDM in fact derive from the "extended" DDM (eDDM; Ratcliff 1978). This variant possesses additional trial-by-trial variability parameters that allow it to recover those empirical effects – while losing the optimality properties that make the pDDM a normatively grounded model (Moran, 2015).

A recently-proposed class of models reframes the conditions under which normative optimality must hold by including the presence of an uncertain deadline, or relaxing the assumption that evidence is temporally independent. These "collapsing-bound" DDM variants replace the fixed decision threshold with a time-varying one, and are argued to achieve both the descriptive advantages of the eDDM and the normative properties of the pDDM (Frazier and Yu, 2008; Drugowitsch et al., 2012; Moran, 2015).

In a recent issue of *The Journal of Neuroscience*, Hawkins and colleagues (Hawkins et al. 2015) attempted to assess the universality of the collapsing bound approach to decisions by focusing on direct fits to data. They assembled nine datasets collected from two species performing several different tasks in order to perform a direct comparison of the descriptive power of the dynamic-threshold approach against the fixed-threshold eDDM.

The authors relied on several simplifying assumptions to constrain their comparisons, at least two of which may bear on their observed results. First, since existing ways of computing normative time-varying bounds rely on computationally-expensive dynamic programming, the authors choose to focus on a general heuristic, the Weibull cumulative distribution function (CDF). This is a three-parameter sigmoid, though for their primary comparison the authors

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allowed only the start and endpoint to vary, while fixing the shape of collapse. This is important because the existing literature differs on the optimal shape, with Frazier and Yu (2008, Fig. 2) reporting concave bounds and Drugowitsch et al. (2012, Fig. 3) convex, under different task constraints. The authors allowed the shape to vary in a secondary analysis, but do not report results from that variant, or any analysis in which shape is the only free parameter.

Second, the authors restrict their comparisons to one fixed-bounds model (the eDDM) and two collapsing-bounds models (Weibull-collapsing-bounds versions of the eDDM and pDDM). The latter comparison is motivated in part by the aforementioned idea that the collapsing bounds and the trial-wise variability in the eDDM's start point and drift rate are providing the same descriptive power targeting long tails and unequal conditional RT distributions. Of interest for rational analysis – one of the strengths of the collapsing bounds DDM – would have been a comparison between the two proven-optimal models (fixed-bound pDDM and dynamic-programming-collapsing-bound pDDM).

To correct for the additional parameters in the collapsed-bound models relative to the others, the authors used the Bayesian Information Criterion (BIC), which adds to the fitted likelihood score a penalty for each additional parameter that is a constant function of the data size. Though standard, BIC is fragile to the parameterization choices of the models under consideration. Because the Weibull CDF is novel in its application to these types of data, the parameters that differ between the compared models had unknown, potentially null, contributions to explanatory power. In these situations it may be more appropriate to apply an adaptive penalty, such as the one provided by the Laplace approximation to the Bayes Factor (MacKay, 2003). It is unclear whether the choice of parameterization and comparison algorithm had meaningful impact on the results provided; any such influence would be in favor of the fixed-bound eDDM.

In the comparison against the collapsing-bound eDDM, the results favor the fixed-bound model, though a noticeable split exists between the human subjects (most of whom are better-fit by fixed bounds) and monkeys (most of whom are better fit by collapsing bounds). In the comparison against the collapsing-bound pDDM, the monkeys were evenly split between the two models, and more humans remained better-fit by the fixed bound model (though not by as large a majority as in the first comparison). Across both comparisons, BIC favors the fixed bounds model overall, though with considerable variability across experiments.

The authors note the apparent divide between model fits to monkey and human behavior. They suggest that the much greater amount of practice in the monkeys could lead to learning to perform the task in a way that is better captured by the collapse models. This is consistent with a simultaneously reported observation by Moran (2015), who notes that setting the relevant decision policy parameters (starting point, drift rate, and bound separation) in these sorts of tasks requires optimization over a large enough space as to likely require extensive learning and feedback. This interpretation could also explain a surprising fit divergence in Hawkins and colleagues' results, between data re-analyzed from an experiment by Palmer et al. (2005) better fit by this collapsing bounds model, and a replication by Hawkins and colleagues (Experiment 1) better fit by fixed bounds: while the Palmer experiment employed extensive pre-training that may have provided the necessary long timescale optimization, Experiment 1 did not.

As alluded to above, the benefits of the collapsed-bound model have been argued only secondarily in its fits to data, and primarily in its ability to provide those fits alongside a normative account of behavior. In sidestepping the latter, the authors lose a potential insight into

selecting an arena for the former. It remains to be seen what would result from a meta-analysis of tasks and conditions under which collapsing bounds have been proven optimal: those with asymmetric priors, heterogeneous and unsignaled difficulty levels, uncertain deadlines, or other variability in trial timing and evidence. It is in those settings where one would expect dynamic bounds to be most distinguishable from the fixed-bound model, under the assertion of rationality.

Future work is also needed to synthesize the disparate conditions under which collapsing bounds are analytically motivated or empirically observed. We see one such synthesis as the class of models where the assumption of i.i.d. evidence samples is broken and therefore the DDM will not inherit the SPRT's optimality. Another approach is to accept collapsing bounds as rational mechanism in the idealized case, but explore representational limitations or costs of performance optimization under which collapsing decision bounds rationally reduce to the fixed case.

With respect to descriptive work agnostic of the rationality question, subsequent demonstrations that the fixed-bound model obtains in direct model comparisons would raise the question of how we should interpret the full empirical picture: should we conclude that previous empirical arguments for time-varying bounds have overfit the data, and the eDDM remains the best consensus model on the strength of this meta-analysis? Should we keep preferring the dynamic-bounds model because the present work did not investigate the best places for it to apply and because it reduces to the fixed-bound model as a special case? Or should we assume that both mechanisms apply in different situations and tasks? Mechanistic parsimony might suggest the first or second option, but the question is an empirical one - different neural circuits could conceivably implement different variations of the same computations and strategically deploy them under different task demands. Recent work has shown that multiple brain regions reflect – if not necessarily always participate in – the evidence accumulation process driving perceptual decisions (Ding and Gold, 2012; Hanks et al., 2015). These structures may operate as discrete elements of a parallel circuit, learning and implementing decision policies with different computational properties. The influence of these policies on behavior may range from negligible to dominant, depending on task.

In sum, Hawkins and colleagues have placed one signpost towards the next-generation consensus theory of the temporal dynamics of decision making, and in doing so highlighted the different contributions that can be made by research groups who differently value explanatory versus descriptive modeling. Further efforts in both directions will be essential in capturing what is optimal about behavior, and suboptimal about our models.

Acknowledgements

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